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Dynamical aspects on duality between SYM and NCOS from D2-F1 bound state

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Abstract: It has been shown that $(2+1)$-dimensional $\mathcal{N}=8$ super Yang-Mills (SYM) theory with electric flux is related to $(2+1)$-dimensional non-commutative open string (NCOS) theory by "$2-11$" flip. This implies that the instanton process in SYM theory, which corresponds to D0-brane exchange(M-momentum transfer) between D2-branes, is dual to the KK momentum exchange in NCOS theory, which is perturbative process in nature. In order to confirm this, we obtain the effective action of probe M2-brane on the background of tilted M2-branes, which would correspond to the one-loop effective action of SYM theory with non-perturbative instanton corrections. Then we consider the dual process in NCOS theory, which is the scattering amplitude of the wound graviton off the D2-F1 bound state involving KK-momentum transfer in x^2-direction. Both of them give the same interaction terms. Remarkably they also have the same behavior on the non-trivial velocity dependence. All these strongly support the duality between those two theories with completely different nature.

Keywords: D-branes, String Duality, M(atrix) Theories.

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1. Introduction

The $(p+1)$-dimensional matrix theory [1, 2] describes the system of N Dp-branes wrapped on the $x^1, \ldots, x^p$ direction in the scaling limit[1]

$$\alpha' \sim \epsilon^{1/2}\,, \qquad g_s \sim \epsilon^{\frac{3-p}{4}}\,, \qquad g_{IJ} \sim \epsilon\,, \tag{1.1}$$

while the metric components in the longitudinal direction are kept fixed in the limit $\epsilon \to 0$. In this scaling limit, all the closed string modes and massive open string modes are decoupled and thus it becomes the theory of massless open string modes only, which is U(N) super Yang-Mills (SYM) theory on the D-branes world-volume. One may note that U(1) multiplet in $\mathrm{U}(N) = \mathrm{U}(1)\times\mathrm{SU}(N)/\mathbb{Z}_N$ degrees of freedom describe the overall motion of the system and are decoupled from the rest degrees of freedom.

Our main concern in this paper is the world-volume theory of Dp-F1 bound states, in which fundamental strings are dissolved into Dp-branes and turn into electric flux on Dp-branes. They are half BPS states just like those of pure Dp-branes. In this case one may again take the same limit as (1.1) which gives ordinary U(N) SYM theory with electric flux. The resultant $\mathrm{SU}(N)/\mathbb{Z}_N$ theory with an $\mathbb{Z}_N$ flux has a mass gap while the remaining U(1) theory is free [3].

One can take different limit of the system, which is, so-called, the NCOS limit [4]. This scaling limit is achieved by considering near critical electric field on Dp-brane:

$$2\pi\alpha'\epsilon^{01}F_{01} = 1 - \frac{\epsilon}{2}\,, \tag{1.2}$$

[1] $\alpha, \beta = 0, 1, \ldots, p$ denote longitudinal directions of the brane. Among them the electric directions on the brane are denoted by $\mu, \nu = 0, 1$ and the remaining directions of the brane are denoted by $i, j = 2, 3, \ldots, p$. $I, J = p+1, \ldots, 9$ denote the directions transverse to the brane. $M, N = 0, 1, \ldots, 9$ denote ten-dimensional coordinates, collectively.

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where critical electric field corresponds to $\epsilon = 0$. The scaling of the background metric for closed string is given by [5]

$$g_{\mu\nu} = \eta_{\mu\nu}\,, \qquad g_{ij} = \epsilon\delta_{ij}\,, \qquad g_{IJ} = \epsilon\delta_{IJ}\,. \tag{1.3}$$

The effective open string tension is $\frac{1}{4\pi\alpha'_e} = \frac{\epsilon}{4\pi\alpha'}$. Therefore, while $\alpha' \equiv l_s^2$ sets the scale of closed string modes, $\alpha'_e \equiv l_e^2$ can be considered as the scale of open string modes stretched in the electric direction.

In the presence of background electric field, or NS B-field on the D-brane world-volume, the effective metric seen by the open strings on the D-brane world-volume is different from the metric seen by bulk closed string modes. The effective open string metric, noncommutativity parameter and effective open string coupling G_o^2 can be determined as [6, 7]

$$G_{\mu\nu} = \epsilon\eta_{\mu\nu}\,, \qquad G_{ij} = \epsilon\delta_{ij}\,, \qquad \Theta^{\mu\nu} = 2\pi\alpha'_e\epsilon^{\mu\nu}\,, \qquad G_o^2 = g_s\epsilon^{1/2}\,. \tag{1.4}$$

The NCOS limit [4], under which the bulk closed string modes are decoupled, is given by $\epsilon \to 0$ while taking α'_e, G_o fixed, and therefore is summarized as

$$g_{\mu\nu} \sim \mathcal{O}(1)\,, \qquad g_{ij} \sim \epsilon\,, \qquad g_{IJ} \sim \epsilon\,, \qquad g_s \sim \epsilon^{-1/2}\,, \qquad a' \sim \epsilon\,. \tag{1.5}$$

In this limit, the effective degrees of freedom are those of open strings on non-commutative spacetime in which

$$[x_0, x_1] = \theta\,.$$

In general, these two limits are connected by U-duality [8]. From the case of D1-F1 bound states with the scaling limits (1.1) and (1.5), one can easily see that the $(1+1)$-dimensional SYM theory on N D-strings with M units flux of electric field is S-dual to $(1+1)$-dimensional NCOS theory of M D-strings with N units flux of electric field [5, 9]. The scaling limits (1.1) and (1.5) imply that the $(2+1)$-dimensional SYM theory with electric flux from D2-F1 bound states wrapped on two-torus is related to (2+1)-dimensional NCOS theory by the, so-called, "$2-11$" flip, i.e. circle compactifications along different directions [5]. Further duality chain on each side relates matrix theory and NCOS theory on higher-dimensional tori [8].

These dual relations between string theories on non-commutative spacetime and ordinary gauge theories, though guaranteed from dualities of "mother" M/string theory, are quite surprising. Immediate question related on these dualities of different type of theories is how to map various kinds of excitations in one theory to those in its dual theory. In this paper, we are especially interested in the dual of stringy degrees of freedom in NCOS theory. In general, massive degrees of freedom of open strings are non-BPS, and thus disappear into multi-particle states of massless spectrum as the coupling becomes strong. In the cases of $(1+1)$-dimensional and $(2+1)$-dimensional theories SYM theories are S-dual to NCOS theories and thus the above arguments may apply. However, in the $(2+1)$-dimensional theory on torus, we have additional BPS states among string spectrum. They are states with KK momentum along compactified x^2-direction. Under "$2-11$" flip, they map to D0-branes, and thus become magnetic flux in the dual SYM theory. The exchange of D0-branes

Theory	NCOS	SYM
(D2, F1)	(M, N)	(N, M)
g_s	$\epsilon^{-1/2} G_o^2$	$\epsilon^{1/4} G_o^{-1} (\frac{r_2}{l_e})^{3/2}$
α'	$\epsilon \alpha'_e$	$\epsilon^{1/2} \frac{G_o^2 l_e^3}{r_2}$
gauge coupling $g_{\rm YM}^2$	$\frac{G_o^2}{l_e}$	$\frac{r_2^2}{G_o^2 l_e^3}$
x^2 radius	$\epsilon^{1/2} r_2$	$G_o^2 l_e$
x^{11} radius	$G_o^2 l_e$	$\epsilon^{1/2} r_2$
	k_2	magnetic flux
	magnetic flux	k_2
l_p^3	$\epsilon G_o^2 l_e^3$	$\epsilon G_o^2 l_e^3$

Table 1: $2+1$-dimensional theories ($\alpha'_e = l_e^2$)

between two D2-branes can be interpreted as the instanton process in $(2+1)$-dimensional SYM theory. In particular, it has been shown that v^4-terms and their superpartners in the effective action of $(2+1)$-dimensional $\mathcal{N}=8$ SYM theory are completely determined by one-loop contribution and instanton corrections [10, 11, 12]. In this paper we focus on the same process in the SYM theory side and consider the dual process in the NCOS theory to confirm the dual relation between instanton process in SYM theory and KK momentum exchange in NCOS theory. In section 2, we obtain the effective action of probe M2-brane on the background of tilted M2-branes, which correspond to the one-loop effective action of SYM theory including non-perturbative instanton corrections. In section 3 we compute the scattering amplitude of the wound graviton off the D2-F1 bound state involving KK-momentum transfer in x^2-direction. In section 4, we draw our conclusions. As for a reference, the relation between these two theories are summarized in table 1.

2. Effective action of probe M2-brane

In this section we consider D2-F1 bound state in the SYM theory limit. In order to obtain the effective action of $(2+1)$-dimensional $\mathcal{N}=8$ SYM theory with electric flux, we use the AdS/CFT correspondence [13, 14], which is well-established in the matrix theory limit [2, 15]. Note that in this limit, the scale of transverse direction and the scale of eleventh direction behave in the same way,

$$r \sim \epsilon^{1/2}\,, \qquad R \sim \epsilon^{1/2}\,,$$

and hence we should take into account the contribution from eleventh direction. D2-F1 bound state becomes tilted M2-branes bound state in eleven-dimensional lift. Therefore we consider probe M2-brane dynamics in the background of source tilted M2-branes in the limit and obtain the effective action of probe M2-brane which corresponds to the one-loop effective action of $(2+1)$-dimensional SYM theory with full non-perturbative instanton corrections.

2.1 Probe dynamics of M2-brane in the AdS_4 background

In this subsection, we review the supergravity dual description of (2+1)-dimensional $\mathcal{N}=8$ SYM theory, without electric flux. In the limit (1.1), the world-volume theory of N D2-branes reduces to (2+1)-dimensional $\mathcal{N}=8$ SYM theory. v^4 terms and their superpartners are completely determined by one-loop and non-perturbative instanton corrections [10]. This is due to the non-renormalization property of the theory with 16 supercharges [11]. On the other hand, the supergravity dual description is given by the probe dynamics of M2-brane in the background of periodically identified AdS_4 spacetime, produced by the background D2-branes in the SYM decoupling limit [12, 16]:

$$ds_{11}^2 = h_0^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + h_0^{1/3}(dx_3^2 + \cdots + dx_9^2 + dx_{11}^2)\,, \tag{2.1}$$

where the eleven-dimensional harmonic function h_0 is given by

$$h_0 = \sum_{n=-\infty}^{\infty} \frac{2^5\pi^2 l_p^6 N}{(r^2 + (x_{11} + 2\pi R n)^2)^3}\,, \tag{2.2}$$

under x^{11}-compactification, $x_{11} \sim x_{11} + 2\pi R$, and $r^2 = x_3^2 + \cdots + x_9^2$. Here N denotes the number of background M2-branes.

We consider the probe dynamics of M2-brane wrapping on x^1, x^2 directions and moving with a constant velocity $v^I = \partial_0 x^I$ and $v^{11} = \partial_0 x^{11}$ in the transverse directions. The bosonic part of the action for the probe M2-brane is given by

$$S_2 = -T_2 \int d^3\xi\, \sqrt{-\det h_{\alpha\beta}} + i\mu_2 \int H\,, \tag{2.3}$$

where $h_{\alpha\beta}$ is the induced metric on the world-volume of the probe M2-brane and is given by

$$h_{\alpha\beta} = \partial_\alpha x^{\hat{M}} \partial_\beta x^{\hat{N}} g_{\hat{M}\hat{N}}\,, \tag{2.4}$$

where $\hat{M}, \hat{N}$ denote full eleven-dimensional spacetime coordinates, $0, 1, \ldots, 9, 11$.

From the configurations we choose, it is natural to use the static gauge in which $x_\alpha = \xi_\alpha$. After plugging the metric (2.1) with the harmonic function h_0 given in (2.2) and expanding in powers of $v^2 = v_I^2 + v_{11}^2$, the action becomes

$$S_2 = \int d^3\xi \left(\frac{1}{2} T_2 v^2 + \frac{1}{8} T_2 h (v^2)^2 + \mathcal{O}\left((v^2)^3\right) \right). \tag{2.5}$$

This effective action contains many informations on the dual SYM theory. The vanishing effective action for $v^2 = 0$ tells that the corresponding configuration is BPS. In the dual SYM theory, 16 supercharge guarantees the non renormalization of kinetic terms, v^2, of bosonic fields, which is consistent with the above action. Furthermore it has been shown that v^4-terms and their superpartners are completely determined by one-loop and non-perturbative instanton corrections. This can be shown to agree with the above action as well, by using Poisson resummation formula:

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} d\phi\, f(\phi)\, e^{2\pi i m \phi}\,, \tag{2.6}$$

on the harmonic function h_0 which becomes

$$\sum_{n=-\infty}^{\infty} \frac{1}{(r^2+(x_{11}+2\pi Rn)^2)^3} = \frac{1}{16R}\left[\frac{3}{r^5}+\frac{1}{r^3}\sum_{m=1}^{\infty}\frac{m^2}{R^2}e^{-mr/R}2\cos\left(\frac{mx_{11}}{R}\right)+\right.$$
$$+\frac{3}{r^4}\sum_{m=1}^{\infty}\frac{m}{R}e^{-mr/R}2\cos\left(\frac{mx_{11}}{R}\right)+$$
$$\left.+\frac{3}{r^5}\sum_{m=1}^{\infty}e^{-mr/R}2\cos\left(\frac{mx_{11}}{R}\right)\right]. \tag{2.7}$$

Note that the first term and the remaining terms with infinite sum over the index m correspond to the one-loop correction and the instanton corrections, respectively, in the effective action of $(2+1)$-dimensional SYM theory.

2.2 The background geometry for D2-F1 bound state

The bound state of D2-branes and fundamental strings is nothing but the tilted M2 branes in eleven dimensions. The eleven-dimensional geometry due to these tilted M2 branes are given by the corresponding $x^2 - x^{11}$ rotation of the above metric (2.1) and [17]:

$$ds_{11}^2 = h^{-2/3}(-dt^2+dx_1^2+dx_2^2)+h^{1/3}(dx_3^2+\cdots+dx_9^2+(dx_{11}-\epsilon^{1/2}dx_2)^2)\,, \tag{2.8}$$

where the harmonic function h becomes

$$h = 1+\frac{2^5\pi^2 l_p^6 N}{(r^2+(\cos\theta x_2+\sin\theta x_{11})^2)^3}\,.$$

Under x^{11}-compactification, $x_{11}\sim x_{11}+2\pi R$, the harmonic function h can be written as

$$h = 1+\sum_{n=-\infty}^{\infty}\frac{2^5\pi^2 l_p^6 N}{(r^2+\sin^2\theta(x_{11}+2\pi Rn-\cot\theta x_2)^2)^3}\,, \tag{2.9}$$

and also can be resummed using Poisson resummation formula as follows:

$$h-1 = \frac{2\pi^2 l_p^6 N}{R\sin\theta}\left[\frac{3}{r^5}+\frac{1}{r^3}\sum_{m=1}^{\infty}\left(\frac{m}{R\sin\theta}\right)^2 e^{-mr/R\sin\theta}2\cos\frac{m(x_{11}-x_2\cot\theta)}{R}+\right.$$
$$+\frac{3}{r^4}\sum_{m=1}^{\infty}\frac{m}{R\sin\theta}e^{-mr/R\sin\theta}2\cos\frac{m(x_{11}-x_2\cot\theta)}{R}+$$
$$\left.+\frac{3}{r^5}\sum_{m=1}^{\infty}e^{-mr/R\sin\theta}2\cos\frac{m(x_{11}-x_2\cot\theta)}{R}\right]$$
$$=\frac{2\pi^2 l_p^6 N}{R\sin\theta}\left[\frac{3}{r^5}+\sum_{m=1}^{\infty}\left(\frac{2}{\pi}\right)^{1/2}\frac{m^2 m^{1/2}}{(R\sin\theta)^5}\left(\frac{R\sin\theta}{r}\right)^{5/2}K_{5/2}\left(\frac{mr}{R\sin\theta}\right)\times\right.$$
$$\left.\times\, 2\cos\frac{m(x_{11}-x_2\cot\theta)}{R}\right]. \tag{2.10}$$

In the $R \to 0$ limit, the geometry becomes the one generated by the bound state of N D2-branes and M fundamental strings:

$$ds^2 = \tilde{f}^{1/2} f^{-1}(-dx_0^2 + dx_1^2) + \tilde{f}^{-1/2} dx_2^2 + \tilde{f}^{1/2}(dx_3^2 + \cdots + dx_9^2)\,, \tag{2.11}$$

where

$$f = 1 + \frac{r_0^5}{r^5}\,, \tag{2.12}$$

Here the rotation angle θ becomes

$$\cos\theta = \frac{g_s M \frac{\sqrt{\alpha'}}{R_2}}{(g_s^2 M^2 \frac{\alpha'}{R_2^2} + N^2)^{1/2}} = \frac{MR}{(M^2R^2 + N^2R_2^2)^{1/2}}\,,$$

and the constant r_0 is given by

$$r_0^5 = 6\pi^2 g_s \alpha'^{5/2}\left(g_s^2 M^2 \frac{\alpha'}{R_2^2} + N^2\right)^{1/2} = 6\pi^2\alpha'^2 \frac{R}{R_2}(M^2R^2 + N^2R_2^2)^{1/2}\,,$$

with x^2-compactification radius R_2.

2.3 Probe M2-brane dynamics in the tilted M2-brane background

In this subsection we would like to get the effective action of probe M2-brane in the background of tilted M2-branes, which is the eleven-dimensional lift of probe D2-brane dynamics in the background of source (D2-F1) bound state as shown in the figure 1.

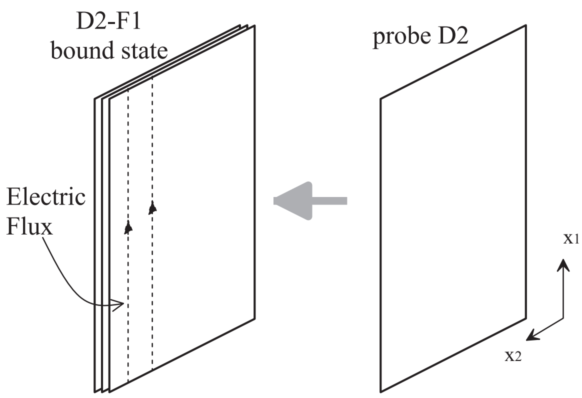

Figure 1: D2 scattering in SYM theory

The probe M2-brane action is given by (2.3), now with new geometry (2.8). In order to recover the results of SYM theory, we need to take the SYM limit (1.1). In this limit, $u = r/\alpha'$ and $\phi_8 = x_{11}/\alpha'$ fixed, and we have[2]

$$\begin{aligned} l_p^3 h = 2\pi^2 N g_{\rm YM}^4 \Bigg[& \frac{3}{u^5} + \frac{1}{u^3}\sum_{m=1}^{\infty}\left(\frac{m}{g_{\rm YM}^2}\right)^2 e^{-mu/g_{\rm YM}^2} 2\cos\left(\frac{m}{g_{\rm YM}^2}\left(\phi_8 - \frac{g_{\rm YM}^2}{R_2}\frac{M}{N}\xi_2\right)\right) + \\ & + \frac{3}{u^4}\sum_{m=1}^{\infty}\frac{m}{g_{\rm YM}^2} e^{-mu/g_{\rm YM}^2} 2\cos\left(\frac{m}{g_{\rm YM}^2}\left(\phi_8 - \frac{g_{\rm YM}^2}{R_2}\frac{M}{N}\xi_2\right)\right) + \\ & + \frac{3}{u^5}\sum_{m=1}^{\infty} e^{-mu/g_{\rm YM}^2} 2\cos\left(\frac{m}{g_{\rm YM}^2}\left(\phi_8 - \frac{g_{\rm YM}^2}{R_2}\frac{M}{N}\xi_2\right)\right)\Bigg] \end{aligned} \tag{2.13}$$

which is finite under the limit $\epsilon \to 0$.

[2]Note that we use static gauge, $x_2 = \xi_2$.

Here ϕ_8 is interpreted as the dual scalar of $(2+1)$-dimensional gauge field A_μ,

$$\partial_\mu \phi_8 = \varepsilon_{\mu\nu\rho} F^{\nu\rho}\,.$$

In the D2-F1 bound state, fundamental strings are dissolved and turned into electric flux, i.e. $F_{01}^{(B)} \neq 0$. This implies the non-trivial background value of the dual scalar $\phi_8^{(B)} \propto \xi_2$ as

$$\partial_2 \phi_8^{(B)} \neq 0\,.$$

This is gauge theory side interpretation why we have extra term in the argument of cosine function in the eq. (2.13), namely, we can set

$$\phi_8^{(B)} = -\frac{g_{\rm YM}^2}{R_2}\frac{M}{N}\xi_2\,. \tag{2.14}$$

The M unit of electric flux in N D2-branes can be expressed as

$$M = N\frac{2\pi R_2}{g_s}\frac{(\alpha')^{1/2}\epsilon^{01}F_{01}^{(B)}}{\sqrt{1-(2\pi\alpha')^2 F_{(B)}^2}}\,. \tag{2.15}$$

Therefore, in the SYM limit, the background electric field is given by

$$F_{01}^{(B)} = \frac{g_{\rm YM}^2}{2\pi R_2}\frac{M}{N}\,,$$

which is consistent with the above assignment (2.14):

The effective action of probe M2-brane in the geometry (2.8) becomes

$$\begin{aligned} S_2 &= -T_2\int d^3\xi\,\frac{1}{h}\left(\sqrt{(1+h\epsilon)(1-h\epsilon\bar{v}_I^2-h\epsilon\bar{v}_{11}^2)+h^2\epsilon^2\bar{v}_{11}^2}-1\right)\\ &= -\frac{T_2\epsilon}{2}\int d^3\xi\,\Big[(1-\bar{v}_I^2-\bar{v}_{11}^2)-\frac{1}{4}h\epsilon(1+\bar{v}_I^2+\bar{v}_{11}^2+2\bar{v}_{11})(1+\bar{v}_I^2+\bar{v}_{11}^2-2\bar{v}_{11})+\\ &\qquad\qquad +\mathcal{O}((h\epsilon)^2)\Big]\,, \end{aligned} \tag{2.16}$$

where h is given by (2.13) and $\bar{v}_I = v_I/\epsilon^{1/2}$ and $\bar{v}_{11} = v_{11}/\epsilon^{1/2}$ are fixed under the limit. One should note that in the present case the effective action is non vanishing even in the case for the vanishing velocity, $v = 0$, as it is not a supersymmetric configuration.

In order to compare with the results of NCOS theory in the next section, we rewrite (2.13) in terms of NCOS variables using the relation shown in table 1 as follows:

$$\begin{aligned} l_p^3 h = 6\pi^2 N\frac{\epsilon^{5/2}G_o^6 l_e^9}{r_2 r^5}&\left[\left(1+\sum_{m=1}^{\infty} e^{-\frac{mr}{r_2\epsilon^{1/2}}}2\cos\left(\frac{mx^{11}}{r_2\epsilon^{1/2}}-\frac{m\xi_2}{r_2}\right)\right)+\right.\\ &+\sum_{m=1}^{\infty}\frac{mr}{r_2\epsilon^{1/2}}e^{-\frac{mr}{r_2\epsilon^{1/2}}}2\cos\left(\frac{mx^{11}}{r_2\epsilon^{1/2}}-\frac{m\xi_2}{r_2}\right)+\\ &\left.+\frac{1}{3}\sum_{m=1}^{\infty}\left(\frac{mr}{r_2\epsilon^{1/2}}\right)^2 e^{-\frac{mr}{r_2\epsilon^{1/2}}}2\cos\left(\frac{mx^{11}}{r_2\epsilon^{1/2}}-\frac{m\xi_2}{r_2}\right)\right]. \end{aligned} \tag{2.17}$$

In the next section we would like to recover the above results from the dual NCOS theory. In particular, we will obtain exactly the same form as the terms linear in the harmonic function h in (2.16).

3. Scattering of fundamental string off D2-F1 bound state in NCOS theory

We now turn to the NCOS theory and consider the process dual to that of D0-brane exchange between D2-branes. The D0-brane exchange is interpreted as the momentum transfer in the eleventh dimension, M-momentum transfer, which becomes the momentum exchange in the x^2 direction in the NCOS theory side, through the "$2-11$" flip or $\mathrm{T_2ST_2}$ duality chain. The probe D2-brane in the SYM side corresponds to a fundamental string in the NCOS theory under the same duality chain. The dual process of the probe D2-brane scattered off the source D2-brane is then given by the usual string amplitude for the scattering of fundamental closed string off the D-brane. The diagram for the amplitude is depicted in figure 2. It has two closed string vertices and, in addition to them, two open string vertex insertions, which are necessary for describing the momentum transfer in the x^2 direction.

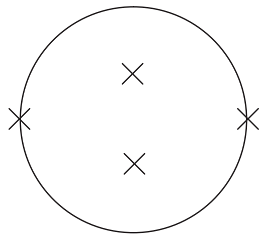

Figure 2: Process in the NCOS theory dual to M-momentum transfer in the SYM theory

The process in the SYM side considered in the last section is the low energy one and assumes that the branes have no fluctuating modes on their worldvolume. This leads us to take the vertices in the string diagram for the dual process to be those for lowest states, that is, massless states. For the closed string, we take graviton states polarized transversely to the D2-brane to consider the process dual to that in the SYM side. Associated with the momentum transfer in the x^2 direction, the vector states polarized along the D2-brane world-volume directions are taken as open string vertices, since the dual process in the SYM side is represented by the gauge field dynamics. Note that the D2-brane carrying electric flux spans x^1 and x^2 spatial directions, which are compactified on circles of radius R_1 and R_2, respectively. According to the duality chain, closed string dual to the probe D2-brane in the SYM side has winding along the x^1 direction. In view of the NCOS limit, this is the right situation, because only closed string winding along the direction where the electric field is turned on (here x^1) can be involved in the NCOS dynamics [18].

For the evaluation of the disk diagram, figure 2, we use the usual doubling trick [19] which expresses the world-sheet field $X(z,\bar{z})$ in terms of its holomorphic part only as follows:

$$X^M(z,\bar{z}) = X^M(z) + (DM^{-1})^M{}_N X^N(\bar{z})\,, \tag{3.1}$$

where D is the diagonal matrix with elements $+1$ in the directions parallel to the D2-brane and -1 in transverse directions. The matrix M is due to the boundary conditions in the presence of the electric field E and its non-trivial part is given by[3]

$$(M^{-1})^\mu{}_\nu = \frac{1}{1-E^2}\begin{pmatrix} 1+E^2 & 2E \\ 2E & 1+E^2 \end{pmatrix}^\mu_{\ \nu}. \tag{3.2}$$

For other directions, M is identity. From now on, we represent DM^{-1} as R for notational convenience;

$$R \equiv DM^{-1}\,.$$

[3] A detailed discussion for the matrices M and D may be found in [20].

The string scattering amplitude we will consider is then given by

$$\mathcal{A} \simeq \int d^2 z d^2 z' \int dy dy' \langle V(z,\bar{z}) V'(z',\bar{z}') V(y) V'(y') \rangle \,. \tag{3.3}$$

where the first two vertices are for wound gravitons[4] and the last two vertices for vector fields on the D2-brane. Each vertex is expressed as follows:[5]

$$\begin{aligned} V(z,\bar{z}) &= G_o^2 \epsilon \xi_{IJ} : V_{-1}^I(p,z) :: V_{-1}^J(R^T\tilde{p},\bar{z}) :, \\ V'(z',\bar{z}') &= \frac{G_o^2}{\alpha'_e} \xi'_{IJ} : V_0^I(p',z') :: V_0^J(R^T\tilde{p}',\bar{z}') :, \\ V(y) &= \frac{G_o}{\sqrt{\alpha'_e}} \zeta \cdot (1+M^{-1})_\alpha : V_0^\alpha(k\cdot(1+R),y) :, \\ V'(y') &= \frac{G_o}{\sqrt{\alpha'_e}} \zeta' \cdot (1+M^{-1})_\alpha : V_0^\alpha(k'\cdot(1+R),y') :, \end{aligned} \tag{3.4}$$

where p and p' ($\tilde{p}$ and $\tilde{p}'$) are momenta of the wound gravitons contributed from the left (right) moving sector of the closed string. The vertex operators in each picture are given by

$$\begin{aligned} V_{-1}^M(q,z) &= e^{-\phi(z)} \psi^M(z) e^{iq\cdot X(z)} \,, \\ V_0^M(q,z) &= \left(\partial X^M(z) + i\frac{\alpha'}{2} q\cdot\psi(z)\psi^M(z) \right) e^{iq\cdot X(z)} \,. \end{aligned} \tag{3.5}$$

The evaluation of the amplitude $\mathcal{A}$ in a fully covariant way is a formidable task and is not our aim. Since we are interested in the process dual to the M-momentum transfer between D2-branes in the SYM side, we now take a particular situation. Firstly, we take the backward scattering at least in the x^2 direction, since, in the SYM side, the D2-branes do not cross each other in the eleventh direction while transferring M-momenta. Secondly, all the momenta are assumed not to have KK momenta in the x^1 direction, which is also the case in the SYM side.

Then, for the open string states, the momenta k and k' are taken to be $k_\alpha = (m/R_2, 0, m/R_2)$ and $k'_\alpha = (-m'/R_2, 0, m'/R_2)$, where m and m' are integers. Since the open string states are massless and thus satisfy $\zeta\cdot k = \zeta'\cdot k' = 0$, and $k^2 = k'^2 = 0$, the polarizations can be chosen to have no components in the x^0 and x^2 directions. The closed string states have winding in the x^1 direction. Two wound gravitons are set to have equal winding number. In the x^2 direction, we take the states to have KK momentum but no winding. The components of each momentum are then as follows: from the left moving part of the closed string, $p_M = (p_0, R_1 w/\alpha', n/R_2, p_\perp)$ ($p'_M = (-p'_0, -R_1 w/\alpha', n'/R_2, -p'_\perp)$) where $p_\perp$ ($p'_\perp$) represents the momentum components transverse to the D2-brane and n, n', and w are integers; from the right moving part, $\tilde{p}_M = (p_0, -R_1 w/\alpha', n/R_2, p_\perp)$ ($\tilde{p}'_M = (-p'_0, R_1 w/\alpha', n'/R_2, -p'_\perp)$). Since the closed string states are wound gravitons which are taken to be polarized transversely to the D2-brane, we have $\xi^T = \xi$, $\mathrm{Tr}\,\xi = \mathrm{Tr}\,\xi' = 0$, $\xi\cdot p = \xi\cdot\tilde{p} = 0$, and $\xi'\cdot p' = \xi'\cdot\tilde{p}' = 0$.

[4]Detailed description for the wound graviton can be found in [21].

[5]The factor $\epsilon \equiv 1 - E^2$ multiplied to the vertex operator in the -1 picture for the wound graviton was absent in a previous literature [22]. This is due to the fact that we use the metric g^{MN} in the Green's function for the world-sheet fermions, eq. (3.7), rather than η^{MN}.

Under the situation taken as above, what we are interested in is the amplitude when the transverse momentum difference between the two wound gravitons is very small, $|p'_\perp - p_\perp| \simeq 0$. It is obtained by looking at the pole terms in $(p'+p)^2$ and its evaluation is performed in a standard way, and thus we will omit the calculational details.[6] However, we would like to note two points: we should bear in mind the momentum conservation law,

$$p + R^T \cdot \tilde{p} + p' + R^T \cdot \tilde{p}' + k \cdot (1+R) + k' \cdot (1+R) = 0\,, \tag{3.6}$$

and the phase factors coming from the space-time noncommutativity do not appear. The latter point implies that the Green's functions for the world-sheet fields that we need are the usual ones, that is,

$$\begin{aligned} \langle X^M(z) X^N(z') \rangle &= -\frac{\alpha'}{2} g^{MN} \ln(z-z')\,, \\ \langle \psi^M(z) \psi^N(z') \rangle &= -\frac{g^{MN}}{z-z'}\,. \end{aligned} \tag{3.7}$$

After fixing the $\mathrm{SL}(2,\mathbb{R})$ symmetry present in the amplitude $\mathcal{A}$ which we choose as $z' = i$ and $y' = \infty$ and doing some manipulations, we obtain the expression for the amplitude as

$$\mathcal{A} \simeq \frac{G_o^4}{\alpha'_e} \frac{1}{\alpha'(p+p')^2} \mathrm{Tr}(\xi \cdot \xi')(\zeta \cdot \zeta') \alpha' k \cdot (p'-p) \alpha' k' \cdot (p'-p) + \mathcal{O}(p'+p)\,, \tag{3.8}$$

where the indices of polarization tensors are contracted with the usual Minkowskian metric η^{MN} while the other contractions are done by using g^{MN}. The part of $\mathcal{O}(p'+p)$ leads to the interactions, more or less, corresponding to the spin-dependent interactions in the dual SYM theory [25]. Such kind of interactions is beyond of our concern because as we are studying the process dual to that in the SYM theory, which is spin-independent.

Now, we plug the components of momenta specified above into $\mathcal{A}$, eq. (3.8), and eliminate p_0 (p'_0) through the mass shell condition $p^2 = 0$ ($p'^2 = 0$). Then, in the NCOS limit, we get

$$\mathcal{A} \simeq \frac{G_o^4}{\alpha'_e} \frac{\mathrm{Tr}(\xi \cdot \xi')(\zeta \cdot \zeta')}{\alpha'_e(\Delta n^2/R_2^2 + q^2)} \frac{R_1^2 m^2 w^2}{R_2^2} (1 + v_\perp^2 + v_2^2 + 2v_2)(1 + v_\perp^2 + v_2^2 - 2v_2) + \mathcal{O}(p'+p)\,. \tag{3.9}$$

where $\Delta n = n' + n$ is the amount of KK momentum exchange in the x^2 direction and $q = p'_\perp - p_\perp$ is the momentum transfer between the closed string and the D2-brane in the transverse directions to the D2-brane. The v's are the "velocities" of the closed string which are defined as

$$v_\perp = \frac{\alpha'_e}{R_1 w} p_\perp\,, \qquad v_2 = \frac{\alpha'_e}{R_1 R_2 w} n\,, \tag{3.10}$$

where $\frac{R_1 w}{r \alpha'_e}$ plays the role of non-relativistic mass [24].

[6] Similar calculation with just transverse momentum ($p_\perp$) transfer has been done in [23], in the context of NCOS theory as well as non-relativistic string theory [24].

Let us turn to the probe dynamics in the SYM theory and expand the potential in term of ϵh. The leading interaction term, which is linear order in h, eq. (2.16), should be compared with the amplitude (3.9). We now see that the velocity factors agree exactly with those in the SYM theory, if we perform replacements, $v_\perp \to \bar{v}_I$ and $v_2 \to \bar{v}_{11}$. As for the interaction type, there should be terms in the NCOS theory that have the same form as those in the harmonic function ϵh in (2.17), as was alluded in the last section. In order to confirm this, we Fourier transform the $1/(\Delta n^2/R_2^2 + q^2)$ factor in eq. (3.9);

$$\begin{aligned}
&\frac{1}{R_2}\sum_{\Delta n} e^{i\Delta n x_2/R_2} \int \frac{d^7 q}{(2\pi)^7} e^{iq\cdot x_\perp} \frac{1}{\alpha'_e(\Delta n^2/R_2^2 + q^2)} = \\
&= \frac{3}{16\pi^3 \alpha'_e R_2} \frac{1}{r^5} \left[\left(1 + \sum_{\Delta n=1}^{\infty} e^{-\Delta n r/R_2} 2\cos\left(\frac{\Delta n x_2}{R_2}\right)\right) + \right. \\
&\qquad + \sum_{\Delta n=1}^{\infty} \frac{\Delta n r}{R_2} e^{-\Delta n r/R_2} 2\cos\left(\frac{\Delta n x_2}{R_2}\right) + \\
&\qquad \left. + \frac{1}{3}\sum_{\Delta n=1}^{\infty} \left(\frac{\Delta n r}{R_2}\right)^2 e^{-\Delta n r/R_2} 2\cos\left(\frac{\Delta n x_2}{R_2}\right)\right] \qquad (3.11) \\
&= \frac{1}{16\pi^3 \alpha'_e R_2} \frac{1}{r^5} \left[3 + \sum_{\Delta n=1}^{\infty} \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{\Delta n r}{R_2}\right)^{5/2} K_{5/2}\left(\frac{\Delta n r}{R_2}\right) 2\cos\left(\frac{\Delta n x_2}{R_2}\right)\right],
\end{aligned}$$

where $r = \sqrt{x_\perp^2}$. This is exactly the same, up to an overall coefficients, as the harmonic function h in (2.17). Note that, in the comparison, "$2-11$" flip has been implied, in which the KK-momentum exchange, Δn, in NCOS theory is traded to the instanton process, i.e. D0-brane exchange, m. This completes the study of process in the NCOS theory dual to that in the SYM theory.

4. Conclusions

The duality between $(2+1)$-dimensional $\mathcal{N} = 8$ SYM theory with electric flux and $(2+1)$-dimensional NCOS theory is inherited from "$2-11$" flip of "mother" M/string theory. Still yet, this duality is quite surprising as those two theories have completely different characteristics. One is the ordinary gauge theory whose low energy excitations are gauge fields, while the other is theory of strings living on non-commutative spacetime. The duality is more or less strong-weak duality. Therefore the general stringy excitations in NCOS theory decay into massless states or stable massive BPS states in the strong coupling limit, and thus can not be seen in the dual SYM theory.

However, we can still find some pieces of evidence of the duality by considering the process involving non-trivial BPS spectrum. One such a process in SYM theory is the one corresponding to D2-D2 scattering process which involves instanton contributions, i.e. D0-brane (or M-momentum, in eleven-dimensional sense) exchange. The dual process in NCOS theory is the KK-momentum transfer in x^2-direction, in the scattering amplitude of the closed fundamental string off the D2-F1 bound state. We showed that both of them give rise to the same results in the linear terms in the harmonic function h, up to overall coefficients. They have the same behavior in the r dependence as well as in the non-trivial velocity dependence. In order to obtain infinite sum of instanton corrections in the dual NCOS theory, we should sum over all the KK-momentum transfer in x^2-direction. Higher order terms in h in the effective action (2.16) are expected to correspond to higher-loop corrections of the same scattering amplitude in NCOS theory.

In $(3+1)$-dimensional case, NCOS theory is S-dual to non-commutative Yang-Mills theory. Furthermore NCOS theory on T^2 is U-dual (T^2ST^2-dual) to $\mathcal{N}=4$ SYM theory with electric flux. Interestingly enough, the gauge coupling in SYM theory is independent of the open string coupling in NCOS theory, and thus it is not clear what happen to the stringy degrees of freedom in the regime of SYM theory. We hope to return this issue in the near future [26].

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